$2.20)$ b) $T_{12}(x_1, x_2, x_3) = \begin{bmatrix} -x_1 + 2x_2 + 2x_3 & \frac{6x_1 - 12x_2 - 3x_3}{4} & -2x_1 + 4x_2 + 4x_3 \end{bmatrix}^T$

$T_{12} : \mathbb{R}^3 \to \mathbb{R}^3$

$T(1,0,0) = [-1 \quad 3/2 \quad -2]^T \to [-1 \quad 3/2 \quad -2]^T_{B\mathbb{R}^3} = [-1 \quad 3/2 \quad -2]^T$

$T(0,1,0) = [2 \quad -3 \quad 4]^T \to [2 \quad -3 \quad 4]^T_{B\mathbb{R}^3} = [2 \quad -3 \quad 4]^T$

$T(0,0,1) = [2 \quad -3 \quad 4]^T \to [2 \quad -3 \quad 4]^T_{B\mathbb{R}^3} = [2 \quad -3 \quad 4]^T$

Por lo tanto: $\boxed{[T_{12}]^{B\mathbb{R}^3}_{B\mathbb{R}^3} = \begin{bmatrix} -1 & 2 & 2 \\ 3/2 & -3 & -3 \\ -2 & 4 & 4 \end{bmatrix}}$

$T_{15}([a \; b \; c]^T) = (a+b) + (a+c)x + (b+c)x^2$

$T_{15} : \mathbb{R}^3 \to \mathbb{R}_2[x]$

$T([1 \; 0 \; 0]^T) = 1 + x \to [1+x]_{B\mathbb{R}_2[x]} = [1 \quad 1 \quad 0]^T$

$T([0 \; 1 \; 0]^T) = 1 + x^2 \to [1+x^2]_{B\mathbb{R}_2[x]} = [1 \quad 0 \quad 1]^T$

$T([0 \; 0 \; 1]^T) = x + x^2 \to [x+x^2]_{B\mathbb{R}_2[x]} = [0 \quad 1 \quad 1]^T$

Por lo tanto: $\boxed{[T_{15}]^{B\mathbb{R}_2[x]}_{B\mathbb{R}^3} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$

$$T_{15}^{-1}(a_0+a_1x+a_2x^2)=\left[\frac{a_0+a_1-a_2}{2}\quad \frac{a_0-a_1+a_2}{2}\quad \frac{-a_0+a_1+a_2}{2}\right]^T$$

$$T_{15}^{-1}: \mathbb{R}_2[x] \to \mathbb{R}^3$$

$$T(1)=\left[\tfrac{1}{2}\quad \tfrac{1}{2}\quad -\tfrac{1}{2}\right]^T \to \left[\tfrac{1}{2}\ \tfrac{1}{2}\ -\tfrac{1}{2}\right]^T_{B\mathbb{R}^3}=\left[\tfrac{1}{2}\ \tfrac{1}{2}\ -\tfrac{1}{2}\right]^T$$

$$T(x)=\left[\tfrac{1}{2}\quad -\tfrac{1}{2}\quad \tfrac{1}{2}\right]^T \to \left[\tfrac{1}{2}\ -\tfrac{1}{2}\ \tfrac{1}{2}\right]^T_{B\mathbb{R}^3}=\left[\tfrac{1}{2}\ -\tfrac{1}{2}\ \tfrac{1}{2}\right]^T$$

$$T(x^2)=\left[-\tfrac{1}{2}\quad \tfrac{1}{2}\quad \tfrac{1}{2}\right]^T = \left[-\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right]^T_{B\mathbb{R}^3}=\left[-\tfrac{1}{2}\ \tfrac{1}{2}\ \tfrac{1}{2}\right]^T$$

Por lo tanto: $\boxed{\left[T_{15}^{-1}\right]^{B\mathbb{R}^3}_{B\mathbb{R}_2[x]}=\begin{bmatrix}\tfrac{1}{2} & \tfrac{1}{2} & -\tfrac{1}{2}\\ \tfrac{1}{2} & -\tfrac{1}{2} & \tfrac{1}{2}\\ -\tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2}\end{bmatrix}}$

b)

$$\left[T_{12}\circ T_{15}^{-1}\right]^{B\mathbb{R}^3}_{B\mathbb{R}_2[x]} = \underbrace{\left[T_{12}\right]^{B\mathbb{R}^3}_{B\mathbb{R}^3}}_{\text{I}} \cdot \underbrace{\left[T_{15}^{-1}\right]^{B\mathbb{R}^3}_{B\mathbb{R}_2[x]}}_{\text{II}}$$

I y II los calculé en a), por lo tanto:

$$\boxed{\left[T_{12}\circ T_{15}^{-1}\right]^{B\mathbb{R}^3}_{B\mathbb{R}_2[x]}=\begin{bmatrix}-1 & 2 & 2\\ \tfrac{3}{2} & -3 & -3\\ -2 & 4 & 4\end{bmatrix}\cdot\begin{bmatrix}\tfrac{1}{2} & \tfrac{1}{2} & -\tfrac{1}{2}\\ \tfrac{1}{2} & -\tfrac{1}{2} & \tfrac{1}{2}\\ -\tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2}\end{bmatrix}=\begin{bmatrix}-\tfrac{1}{2} & -\tfrac{1}{2} & \tfrac{5}{2}\\ \tfrac{3}{4} & \tfrac{3}{4} & -\tfrac{15}{4}\\ -1 & -1 & 5\end{bmatrix}}$$

Busco núcleo:

$$\begin{cases} -\frac{x_1}{2} - \frac{x_2}{2} + \frac{5}{2}x_3 = 0 \\ \frac{3}{4}x_1 + \frac{3}{4}x_2 - \frac{15}{4}x_3 = 0 \\ -x_1 - x_2 + 5x_3 = 0 \end{cases}$$

$$\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \\ \frac{3}{4} & \frac{3}{4} & -\frac{15}{4} \\ -1 & -1 & 5 \end{pmatrix} \begin{matrix} \\ F2 \to \frac{3}{2}F1 + F2 \\ F3 \to F1 - \frac{1}{2}F3 \end{matrix} \quad \begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to -\frac{x_1}{2} - \frac{x_2}{2} + \frac{5}{2}x_3 = 0 \quad \text{I}$$

$$\text{I} \to \frac{x_1}{2} = -\frac{x_2}{2} + \frac{5}{2}x_3 \to x_1 = -x_2 + 5x_3$$

$\bar{X}$ que cumplen $\to$ $\bar{X} = (-x_2 + 5x_3, x_2, x_3) = x_2 \cdot (-1, 1, 0) + x_3 \cdot (5, 0, 1)$

Por lo tanto una base del núcleo sería: $\boxed{\{[-1,1,0],[5,0,1]\}}$